DESIGNED AND EMERGENT PEDAGOGICAL SUPPORTS FOR COORDINATING QUANTITATIVE AND AGENT-BASED DESCRIPTIONS OF COMPLEX DYNAMIC SYSTEMS

Michelle Hoda Wilkerson-Jerde
Tufts University
Michelle.wilkerson@tufts.edu

Uri Wilensky
Northwestern University
uri@northwestern.edu

Often, quantitative and mathematical models are used to study complex dynamic systems–where collections of individual components interact to produce an emergent pattern of behavior. To interpret such models involves 'unpacking' the collective behaviors and interactions that underlie a given pattern. We investigate what representational supports in a technology-mediated agent-based modeling environment led secondary students to question and investigate the relationship between individual behaviors and collective patterns in a population dynamics unit. Our goals are to inform 1) our own design of educational tools and 2) the theoretical elaboration of general "conceptual leverage points" for mathematics and complex systems education.

Objectives / Purpose

Over the past several decades, there have been a number of technological and theoretical developments that have enabled scientists to explore the interconnected and complex nature of natural and social phenomena in ways that were not previously possible (Bar-Yam, 1997; Mitchell, 2009). This increasing capacity to document and study the world's complexity has led calls for complex systems topics and principles to be integrated into the K-16 curriculum (Forrester, 1994/2009; Jacobson & Wilensky, 2006; Kaput et al., 2005; Sabelli, 2006). Indeed, in her plenary session at PME-NA last year, English noted that technology and complexity "have led to significant changes in the forms of mathematical thinking that are required beyond the classroom" – and that technology has increased the demand for "the interpretation of data and communication of results" (2010, p. 33).

Our objective is to better understand how we can support students as they engage in one such "new" form of mathematical thinking – to interpret mathematical and quantitative models in terms of the complex, interactional events that they encapsulate and measure. This report is part of a larger design-based research project (Cobb, Confrey, diSessa, Lehrer, & Schauble, 2003) that seeks to (1) explore student reasoning about complex dynamic systems and associated mathematical ideas including the measurement and encapsulation of complex behavior, notions of rate of change and accumulation in multivariate systems, and nonlinearity; and (2) develop educational tools and activities that support such reasoning.

Background and Theoretical Framework

One important form of mathematical thinking related to complex systems involves understanding the relationship the behavior of a system at different *levels* (Wilensky & Reisman, 2006). Often, the relationships between the *individual* and *collective* levels of a system can seem inconsistent, since many individual behaviors and interactions underlie a given collective result – for example, individual cars in a traffic jam each move forward even as the jam itself propagates backward; and chemical reactions occur at the atomic level even as chemical concentrations

Wiest, L. R., & Lamberg, T. (Eds.). (2011). Proceedings of the 33rd Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education. Reno, NV: University of Nevada, Reno.

reach equilibrium (Wilensky & Resnick, 1999). Similarly, it is common for information about complex systems to be represented in dynamic quantitative and mathematical forms at the aggregate level, even while their underlying individual behaviors are not described mathematically (Blikstein & Wilensky, 2009; Stieff & Wilensky, 2003; Wilensky, 2003). Hence a primary issue in the interpretation and communication of data and mathematical models of complex systems is to infer and reason about what behaviors they may encapsulate (Author, 2010).

One way to encourage students to interpret mathematical behavior is by actually enabling them to simulate and model the phenomena that underlie it. Two decades of work has explored how this can be done with dynamic phenomena such as motion (Kaput, 1994; Nemirovsky, Tierney, & Wright, 1998; Stroup, 2002), banking (Wilhelm & Confrey, 2003), and other physical scenarios (Carlson, Jacobs, Coe, Larsen, & Hsu, 2002; Yerushalmy, 1997). This is not limited to systems where patterns are generated by and reflect a single causal mechanism, but also collections of mechanisms – Confrey and Smith have shown that multiplicative reasoning can be supported by considering individual elements in a collection each "splitting" to create exponentially more entities (1995). Doerr has shown that even the relationship between probabilistic collective behavior and theoretical idealized models can be explored by students by juxtaposing theoretical and empirical models (2000).

We are exploring the potential for *agent-based modeling* (ABM) environments to provide a flexible context for students to learn about quantitative / mathematical patterns that represent complex dynamic systems. In an agent-based model, individuals that comprise a given complex system are simulated by computational *agents* whose behavior and interactions are governed by simple rules. Each agent executes these rules iteratively to simulate a collective emergent outcome, which can be expressed both visually and quantitatively with numbers and plots. While ABM has been shown to help students make sense of the relationship between individual and aggregate behaviors in many *scientific* domains (Jacobson & Wilensky, 2006), we argue that it can also be used to support students as they explore trends represented via data and mathematical models (Authors, 2009).

Of course, any multirepresentational environment is successful only insofar as its representations relate to educationally relevant tasks and students' existing knowledge (Goldman, 2003; Kaput, 1998). To the extent that complex systems are described at multiple levels using multiple representational systems, we argue that it is very much an educationally relevant task to students to navigate and coordinate those descriptions and representations. In this paper, we concern ourselves with supporting students in accomplishing this task by exploring how well the representations that students interact with while engaging in agent-based modeling resonate with and build upon their existing knowledge.

Research Question

We are interested in how students' interactions with agent-based modeling and associated representations can help them make sense of the relationship between a collective, quantitative pattern exhibited by a complex system, and the individuals, behaviors, and interactions that pattern encapsulates. Toward this end, we utilize design-based research to answer the question: <u>What representational supports made available through our agent-based modeling activities led students to *attend to* the particularities of the encapsulated individual-to-collective relationship in complex systems, and what supports do they use to *resolve* the relationship that leads to those particularities in population dynamic systems?</u>

Methods and Design

Participants

We draw our data from two iterations of a 200-minute classroom-based educational intervention implemented across six classrooms at two public metropolitan high schools in the Midwestern United States. “Preparatory High” (PH) is an academically selective senior high school with a student population of over 1,000. At the time of our study, roughly one-third of PH students were identified as low-income, about 40%, 30%, 25%, and 5% as White, Asian, Hispanic, and Black/Native American/Other, respectively, and over 98% of students met or exceeded state standards. We conducted our intervention in two AP Biology classes, each which included between 20 and 25 students from grades 10 through 12 who were simultaneously enrolled in Precalculus, Calculus, and/or Statistics courses. “Local High” (LH) is an open enrollment senior high school with student population of less than 200. At the time of our study, 99% of LH students were identified as low-income, 100% as Hispanic, and about 25% met or exceeded state standards. We conducted our intervention in four Precalculus classes at LH, each having between 15 and 25 students from grades 10 through 12. Both schools had high numbers of students that met or exceeded expected levels of growth as predicted through state standardized exams.

Design and Activities

Our implementation included the use of a *modeling toolkit* and *associated activities*, each which reflect our design hypothesis regarding what aspects of the environment may attune students to the complex connection between individual and collective behavior. Though it is beyond the scope of this paper to review our design in detail, it was heavily informed by projects including SimCalc Mathworlds (Roschelle & Kaput, 1996), Algebra Sketchbook , Modeling4All (Kahn, 2007), Agentsheets (Repenning & Ambach, 1997), Trips (Clements, Nemirovsky, & Sarama, 1995), Function Probe (Confrey & Maloney, 1996), My Graph Rules (Wilensky & Abrahamson, 2006), and various population dynamics activities (Blanton, Hollar, & Coulombe, 1996). The three supports emphasized by these tools and activities that we examine in this paper include the *graphical noise* produced by agent-based models, the *behavioral descriptions* used to create the models, and *spatiotemporal visualization* of agent behavior in models synchronized with plots of the quantitative patterns that emerge from their execution.

The Modeling Toolkit

The modeling toolkit utilized by students in our study consisted of new construction and analysis modules for the NetLogo agent-based modeling environment (Wilensky, 1999). The first module, DeltaTick (Figure 1), enables users to rapidly construct computational agent-based models from a library of visual programming blocks that each represent behaviors, constraints, or quantities that may be of interest when modeling quantitative trends within a specific domain of inquiry. For the population dynamics activities described in this paper, we used a library that provided students with agent instructions such as “reproduce” and “die” with certain probabilities, “wander” around a spatial world, and constrain their reproductive and death behavior based on environmental or individual “if” statements (concerning the availability of space or partners or an agent’s age) factors. The full set of “behavior blocks” available for students to construct models from is featured below. The second module, HotLink Replay (Figure 2), enables users to replay a simulation, and interact with enhanced plots that enable the user to click on a given graphical feature and observe that point of time in the visualization of the simulation, to overlay and compare graphs across simulations, and to zoom in and annotate

specific regions.

The Classroom Activities

Over the course of each 200-minute implementation, students completed a set of activities designed to help them recognize and build connections across individual and quantitative aggregate behavior in population dynamic models. For the purposes of this paper we focus our analysis on two activities that were completed at both sites: the “Probabilistic to Exponential” activity and the “Make Graph Fit” activity. In
“Probabilistic to Exponential”, activity students were guided through the construction of a simple simulation wherein 100 computational agents each have a .01 chance of reproducing

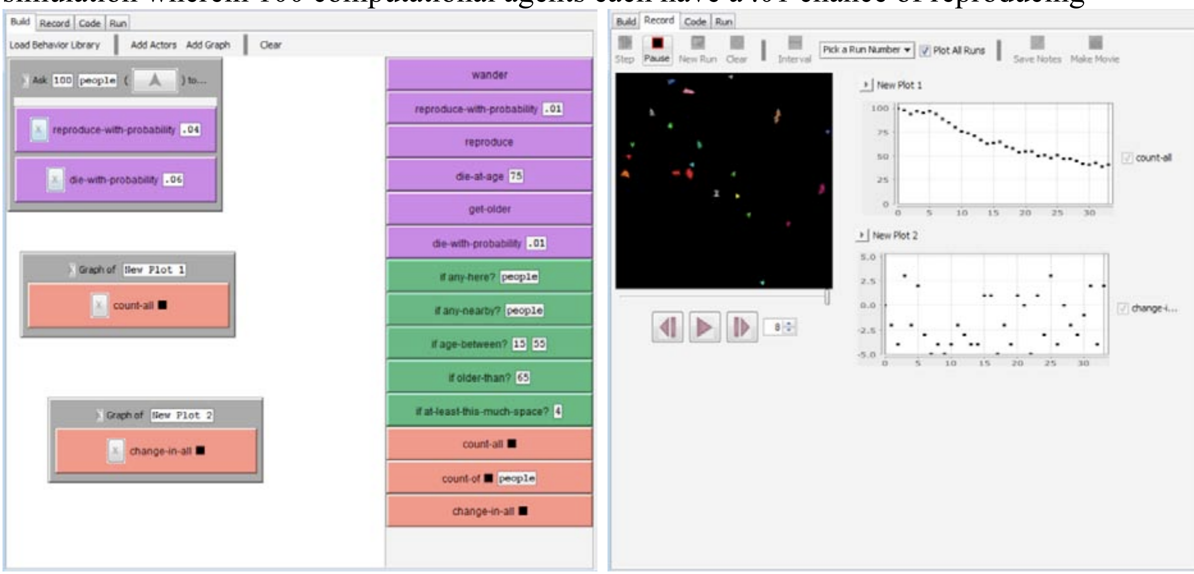

Figure 1. The *DeltaTick* (left) and *HotLink Replay* (right) modules.

per unit of time, and plots of both the number of births and the total number of people per iteration of time were tracked. They were then asked questions designed to emphasize the connection between the “noisy” data and the standard exponential population growth model. In the “Make Graph Fit” activity, students were provided a collection of population patterns including exponential decay, relative stability, logistic-like growth, linear-like growth, and density-induced growth (whereby individuals reproduce only when spatially near others), and asked to find combinations of behaviors that recreated each pattern.

Data Collection

During each implementation, groups of 2 to 3 students worked on laptops equipped with Camtasia Screen Recorder Software, which enabled us to capture and synchronize students’ on-screen activity with their group discussions as captured by video cameras mounted within each laptop computer. These captured videos were then segmented by activity sequence and transcribed using Inqscribe Transcription Software. Since for the purpose of this paper we are interested in students’ interaction with the designed tool and the role that activity prompts played in guiding students to attend to the connections between individual behavioral and aggregate quantitative representations of population dynamics, we focus our current analysis on the collection of video data segments that capture the Probabilistic to Exponential and Make Graph Fit activities.

Wiest, L. R., & Lamberg, T. (Eds.). (2011). Proceedings of the 33rd Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education. Reno, NV: University of Nevada, Reno.

Analytic Approach

To explore our primary research question, we coded each activity segment video to for the students in a given group *questioned* a representational support or *attended to* the fact that a support illustrated a particularity between individual and collective system behavior, and *resolved* to actually explore and better understand that relationship. By "explore and better understand the relationship", we mean attempting to resolve the particularity. We attended both to supports that we hypothesized would emphasize individual-to-collective connections (graphical noise, behavioral enactment, and temporal visualization) as well as unexpected supports that emerged from our data in order to pursue the dual goals of testing our existing hypothesis regarding how agent-based modeling can engage and support students in reasoning about the "individual-to-collective" relationship, while also identifying other potential supports that represent additional pathways for learning (Cobb et al., 2003).

Results

Since we are interested in exploring our hypothesized supports as well as identifying those that emerged from students' interactions with our designed environment, we present our results in two sections. First, we describe students' interactions with our three hypothesized supports for bridging quantitative to individual differences: graphical noise, behavioral enactment, and spatiotemporal visualization. Second, we briefly review other supports that students leveraged to question and begin to interpret the relationship between individual probabilistic and collective quantitative behavior.

Designed Supports for Bridging Quantitative to Individual Descriptions Graphical Noise

We expected graphical noise to alert students to the complex relationship between individual behavior and collective quantitative trends by emphasizing the probabilistic nature of the underlying behavior that graphs measured. Specifically, during the "Probabilistic to Exponential" activity, we guided students to construct a graph measuring the *change in population* to emphasize that the probabilistic nature of individual agents' reproduction manifested quantitatively. We found that while nearly half (7 out of 16) of the groups explicitly questioned the noise produced by the *change in population* graph even before they were prompted to do so by the accompanying activities, only one group found this to be enough motivation to explore the issue in more depth. When explicitly prompted to explain the noise in the accompanying activities, many groups did in terms of the relationship between change and accumulation in the available graphs, without connecting to probabilistic agent behavior.

Initial Questioning 7/16 groups	Change Explanation 1/16 groups no prompting 7/16 groups prompting	Probabilistic Explanation 3/16 groups prompting
"Like, it's going up and down..." "Wow, why is it doing that? … Is this derivative? I don't think so… Why is it crazy?" "Like, it's...the graph is kind of like random"	Ana: I don't get the random dots. Jorge: They're not random. It's just...it's showing you the...let it get to 150 and I'll.. Alright let's go right here (zooms in to plot of change). 175 (zooms in to plot of population). It's just the change in one relation. So like from here...(adjusts graphs). Do you get what I'm saying, though?	"It says sometimes the graph of change-in-all goes down, even though the population is always growing. Because there's still probability that they might not have any...that they might not reproduce at all."
Other groups described the noise as the result of death, (1), an inherent feature of the program itself (1), or could not be classified (3).		

Individual Behavioral Enactment/Programming

A second factor that we expected to alert and support students in attending to to the connection between individual behavior and collective quantitative patterns was that during both activities, students engaged in the programming of individual rules as behavioral descriptions. We expected that when reasoning about the quantitative patterns generated by those models, students would recall either generic descriptions of individual behaviors that contribute to population growth (for example, notions of "having babies", "dying", or "counting people") or reference to the specific individual instructions they programmed ("one guy reproduces by 1%", "die with probability .05". Unlike the noise revealed in graphs, students did not express that the behaviors they programmed into the simulation and the resulting outcomes were inconsistent or unexpected. However, when they were asked to explain what individual behaviors contributed to the resultant trend in the "Probabilistic to Exponential" activity, they often did in only a generic form "change in population", "reproduction". It was not until the "Make Graph Fit" activities that students described trends in terms of the specific rules they programmed "reproduce by 1%", "die with probability .05". While it is not surprising that students would refer to the rules they used when working to build models in order to interpret their resultant behavior, this does illustrate the potential for construction activities to emphasize the multivariate, "encapsulative" nature of patterns in complex systems.

	Generic Explanations **8/16 groups Prob to Exp** **5/13 groups Make Graph Fit**	**Rule-Based Explanations** **3/16 groups Prob to Exp** **8/13 groups Make Graph Fit**
Probabilistic to Exponential	"I was thinking that maybe this relates to this, because it's actually telling you how many times they actually reproduce, which helps you understand what's going on..."	"Because individually they're still subject to reproduce by 1%. Over the long run, though, it increases..."
Other groups provided change-based descriptions (2) or could not be classified (3).		
Make Graph Fit	Herman: Give them their space. You can't reproduce that much. Eric: Alright...it's falling down. Janet: It's not growing fast enough.	John: It's not going to like replace though. Because if one guy reproduces, he'll make one new person. Whereas, if, if it's .99, then 99 people die. Franco: No. John : They wouldn't replace each other, though. Franco: Chance too high. John: They just die.

Spatiotemporal Visualization

Finally, we expected the side-by-side and synchronous nature of a spatiotemporal, agent-based visualization and a quantitative plot to emphasize that those quantitative patterns emerged from counting the addition and aggregation of individual agents. We found that few groups attended to this relationship during the "Probabilistic to Exponential" activity (only 2/16 groups mentioned any features of the visualization, and one group only did so to note that they hadn't paid attention to the visualization). In the "Make Graph Fit" activity, as students built models that featured spatial and interactive behaviors, they relied more on visualization to understand how those features led to patterns such as logistic growth (5/13 groups observed whether their behaviors played out in visualizations).

Emergent Supports for Bridging Quantitative to Individual Descriptions

Questioning Measurement of Behavior. The first, and perhaps the most impressive, support that led students to critically examine the relationship between simulated individual behaviors and the quantitative patterns those behaviors produced was to question what, exactly, each quantity was measuring. In our design, we expected that asking students what agent behaviors would *change* population would lead students to view population as encapsulating those behaviors. Instead, the results above imply that they may have seen the two factors as *related* without exploring whether they were explicitly causal.

Modeling Assumptions

A second support that led students to examine the relationship between individual and quantitative behavior was to question the assumptions about human behavior that underlied a given set of programmed rules. This manifested itself dramatically when one group of students questioned whether agents exhibited a 9 month gestation period or reflected any assumptions about a given demographic populations' behavior. The group then decided that in order to better understand the underlying assumptions of the model, they would conduct experiments with a

single individual under different rules. This led them to consider how that single individual would influence a population.

The Reversibility Script

Finally, several of the students from our first research site, PH, used clues from the questions we asked in our activity set (including questions regarding the relationship between change and population graphs, and regarding the meaning of "change") to infer that it was a calculus-based reversibility relationship, rather than an individual-to-collective relationship, that was the main goal of their exploration. While these two concepts are not mutually exclusive, this "graphical/quantitative reversibility" appeared to supplant students' potential exploration of individual-to-collective behavior using the environment.

Discussion

We sought to identify what designed and emergent supports can help students to understand and navigate the relationship between quantitative patterns generated by complex systems, and the individual behaviors that underlie those patterns. We found that three supports – graphical noise, behavioral enactment, and spatiotemporal visualization, each played differential roles in drawing students' attention to and supporting their investigation of this relationship. We also identified emergent supports – specifically, the potential for framing graphical noise in the context of questioning what, exactly, is being measured; engaging students in discussions about the assumptions that underlie a given agent-based or mathematical model; and *downplaying* the reversibility of graphs until connections to the underlying behavior of a model is established, as potential paths forward in our development of computational modeling tools to support students as they engage in the new forms of mathematical thinking that are required in an age of complex systems.

References.

Bar-Yam, Y. (1997). *Dynamics of complex systems*. Reading, MA: Addison-Wesley.

Blanton, M. L., Hollar, J. C., & Coulombe, W. N. (1996). Calculus students' graphical constructions of a population growth model. *The Mathematics Educator, 7*(1), 15-25.

Blikstein, P., & Wilensky, U. (2009). An atom is known by the company it keeps: A constructionist learning environment for materials science using agent-based modeling. *Intnl. Journal of Computers for Mathematical Learning, 14*(2), 81-119.

Carlson, M., Jacobs, S., Coe, E., Larsen, S., & Hsu, E. (2002). Applying covariational reasoning while modeling dynamic events: A framework and study. *Journal for Research in Mathematics Education, 33*(5), 352-378.

Clements, D., Nemirovsky, R., & Sarama, J. (1995). Trips. White Plains, NY: Dale Seymour.

Cobb, P., Confrey, J., diSessa, A., Lehrer, R., & Schauble, L. (2003). Design experiments in educational research. *Educational Researcher, 32*(1), 9-13.

Confrey, J., & Maloney, A. (1996). Function probe. *Communications of the ACM, 39*(8).

Confrey, J., & Smith, E. (1995). Splitting, covariation, and their role in the development of exponential functions. *Journal for Research in Mathematics Education, 26*(1), 66-86.

Doerr, H. M. (2000). How Can I Find a Pattern in This Random Data? The Convergence of Multiplicative and Probabilistic Reasoning. *Jnl. of Mathematical Behavior, 18*(4), 431-454.

English, L. D. (2010). *Promoting student understanding through complex learning.* Paper presented at the 32nd annual meeting of PME-NA, Columbus, OH.

Forrester, J. W. (1994/2009). *Learning through system dynamics as preparation for the 21st century*. Paper presented at the Systems Thinking and Dynamic Modeling Conference for K-12 Education, Concord, MA.

Goldman, S. (2003). Learning in complex domains: When and why do multiple representations help? *Learning and Instruction, 13*, 239-244.

Jacobson, M., & Wilensky, U. (2006). Complex systems in education: Scientific and educational importance and implications for the learning sciences. *Journal of the Learning Sciences, 15*(1), 24.

Kahn, K. (2007). *Building computer models from small pieces.* Paper presented at the Proceedings of the 2007 Summer Computer Simulation Conference, San Diego, CA.

Kaput, J. (1994). Democratizing access to calculus: New routes to old roots. In A. Schoenfeld (Ed.), *Mathematical thinking and problem solving* (pp. 77-156). Hillsdale: Erlbaum.

Kaput, J. (1998). Representations, inscriptions, descriptions and learning: A kaleidoscope of windows. *Journal of Mathematical Behavior, 17*(2), 265-282.

Kaput, J., Bar-Yam, Y., Jacobson, M., Jakobsson, E., Lemke, J. L., & Wilensky, U. (2005). Planning documents for a national initiative on complex systems in k-16 education.

Mitchell, M. (2009). *Complexity: A guided tour*. New York, NY: Oxford Univeristy Press.

Nemirovsky, R., Tierney, C. C., & Wright, T. (1998). Body motion and graphing. *Cognition and Instruction, 16*(2), 119-172.

Repenning, A., & Ambach, J. (1997). The agentsheets behavior exchange: Supporting social behavior processing. *CHI '97, Conference on Human Factors in Computing Systems*, 26-27.

Roschelle, J., & Kaput, J. (1996). Simcalc mathworlds for the mathematics of change. *Communications of the ACM, 39*(8), 97-99.

Sabelli, N. (2006). Complexity, technology, science, and education. *Journal of the Learning Sciences, 15*(1), 5-9.

Stieff, M., & Wilensky, U. (2003). Connected chemistry--Incorporating interactive simulations into the chemistry classroom. *Journal of Science Education and Technology, 12*(3), 285-302.

Stroup, W. M. (2002). Understanding qualitative calculus: A structural synthesis of learning research. *International Journal of Computers for Mathematical Learning, 7*, 167-215.

Wilensky, U. (1999). NetLogo: http://ccl.northwestern.edu/netlogo.

Wilensky, U. (2003). Statistical mechanics for secondary school: The gaslab modeling toolkit. *International Journal of Computers for Mathematical Learning, 8*(1), 41.

Wilensky, U., & Abrahamson, D. (2006). *Is a disease like a lottery? Classroom networked technology that enables student reasoning about complexity*. Paper presented at the American Educational Research Association, San Francisco, CA.

Wilensky, U., & Reisman, K. (2006). Thinking like a wolf, a sheep or a firefly: Learning biology through constructing and testing computational theories - An embodied modeling approach. *Cognition and Instruction, 24*(2), 171-209.

Wilensky, U., & Resnick, M. (1999). Thinking in Levels: A Dynamic Systems Perspective to Making Sense of the World. *Journal of Science Education and Technology, 8*(1).

Wilhelm, J. A., & Confrey, J. (2003). Projecting rate of change in the context of motion onto the context of money. *International Journal of Mathematical Education in Science and Technology, 34*(6), 887-904.

Yerushalmy, M. (1997). Mathematizing verbal descriptions of situations: A language to support modeling. *Cognition and Instruction, 15*(2), 207-264.

Wiest, L. R., & Lamberg, T. (Eds.). (2011). Proceedings of the 33rd Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education. Reno, NV: University of Nevada, Reno.